

UNCONVENTIONAL METHOD FOR ESTIMATION OF OIL-RESERVES RECOVERY FACTOR AND TIME USING RATE DECLINE TRENDS ANALYSIS

Dr. C. A. Udie

Petroleum Engineering Department
SEET, Federal University of Technology Owerri
(FUTO) Nigeria

Dr. M. S. Nwakaudu

Chemical Engineering Department
SEET, Federal University of Technology Owerri
(FUTO) Nigeria

ABSTRACT:

High accuracy evaluation models for predicting oil-reserves, recovery-factor and time were successfully developed, using production rate decline trend analysis. Existing oil production data from wells in the Niger Delta geological formations (Agbada, Akata and Benin) were used to generate decline constants 'b' that were subsequently used in predicting yearly production data for any given period. The yearly data obtained were validated using the actual yearly production records of the original data source. The validated yearly data were used to generate evaluation curves. The evaluation models were subsequently worked out from the shape of the generated curves. The models were then used to estimate reserves (cumulative and initially in place) in each of the reservoirs. The values obtained compared favorably with the respective storage tank and the volumetric materials balance equations values. The percentage accuracy for oil ranged from 98.64% to 99.98%. The results of this research simplifies complex simulation methods, improves dynamic fluids computational analysis, reduces time in the conventional decline analysis and makes it easy to identify dominated flow and rates decline trends. The models are very flexible and can be applied with high accuracy from the reservoir decline stage to abandonment. They are equally used to estimate the remaining reserves based on the time differences between final and production ($t_f - t_p$) and for the establishment of production and economic decisions techniques.

KEYWORDS: Unconventional Gas Reserves Estimation, Rate Decline Trends, Rate Decline Constant, Projectile and Parabolic Methods

INTRODUCTION:

DEFINITION:

Decline Curve Analysis is a procedure to study reserves recovering rates, using production data or history, based on mathematical equations, tabulated values and graphical representation. Or Decline Curve Analysis is a Curve-Fitting & Extrapolation Method Where, Sample curves are matched-up Standard curves generated with regional data. Reserves prediction is by extrapolation of the matched samples curve to desired points.

BACKGROUND INFORMATION:

There are no fundamental theoretical trends for decline curves analyses, but the exercise is based on production data trend. For this the principal challenge is to minimize errors. All data must be understood before use. There are three principal types of decline rate as postulated by the early researcher. These are exponential or constant decline rate, harmonic decline rate and hyperbolic decline rate. This classification is based on constant or variable changes in the factors that influence the fluid flow in a porous medium. The equation of a fluid flow through porous media under boundary conditions is based principally on steady-state, semi-steady state and unsteady-state and are applied as deemed fit for any particular situations single or two phase fluid stream. Any stream can exhibit any type of decline rate. It depends on the influencing factors. The practical approach to gas production decline rate analysis is to choose the variable such as gas

which results in a reasonable trend. The decline rate trends are used to predict the future well performances. The accuracy in predicting the future gas stream performance depends on the ability to understand the reservoir

characteristics and the standard established for estimating the reserves. In this case best rate decline trends analyses would be compared with volumetric calculated values, MBE values and recovery factor values. The decline curves analysis results will be the estimation tools for the cumulative hydrocarbons production and hydrocarbons initially in place. Field records showed that recoverable hydrocarbons are affected by the operating conditions. When a well is placed on production, there will be transient flow initially, because the boundary conditions are not active enough. Eventually the reservoir boundaries would be felt and it is only then that decline rate becomes clear to predict the value of the decline rate constant (b). It is very useful to have production decline rate model in the Niger Delta and other fields in order to predict projected production rates and estimate both reserves in place and the recovery factor in a reservoir. This equally defines the production decline trend and the process that starts a transient state, peak and decline to minimum level or abandonment rate. The decline models would enable a prediction of the recovery efficiency profile, gives the investors much knowledge of his business profile or trend. Many reserves are abandoned early, because of complex simulation procedures in order to establish motivated economic techniques. Conventionally, volumetric material balance equations (MBE) methods in use are limited to static conditions of the reservoirs and less accurate in the dynamic fluids computation analysis. Equally conventional decline analysis is less accurate, because most researchers assumed exponential or constant rates decline. In reality some reservoirs are not. In this work, mathematical equations or relationships are developed to increase DFCA accuracy. To justification this study, it is necessary to simplify the complex simulation procedures in the conventional methods for rate decline analysis. This would increase DFCA accuracy, reduce the simulation complexity and time used. The success of this work will give an investor the view of his business and it improves his decision on the business. This work primarily covers production decline rates characterization for some gas wells in the Niger Delta. The collated data covered the unsteady-stage (early-stage), steady-stage and semi steady-stage (decline-stage) of a reservoir. The complete production data to abandonment can be used for mathematical equations derivations and confirmation.

REVIEW OF THE SIMULATION AND MODELING IN RESERVES ESTIMATION:

Arps, (1945)^[1] used an empirical relationship and analyzed hydrocarbons production decline curves. In his work he defined hydrocarbons production decline rate as a fractional change (a) in the flow rate (q) with respect to time (t). His mathematical equations are: $a = \frac{-dq/dt}{q_i}$, stb/d or stb/yr and $N_p = \frac{q_i - q}{a}$

2.1

CRAFTS AND HAWKINS, (1959)^[2] field records showed that In decline curve analysis it is implicitly assumed that factors causing the historical decline in a fluid stream would continue unchanged throughout the forecasting period and these factors are the reservoir and operating conditions. The flow rate was plotted against time to predict projection rates and the daily gas production was plotted against time to estimate future cumulative production and reserves originally in place. The most convenient dependent variable is the rate, because extrapolation of the rate-time graph was used directly to forecast the fluid production and economic evaluations. Plots of rate against daily gas production rate equally provided direct ultimate recovery at a given economic limit and yielded a more rigorous interpretation where the production was influenced by intermittent operations.

KATZ, D. L., (1959)^[3] "Handbook of Natural Gas Engineering" McGraw-Hill, Inc., New York.

Arps, (1962)^[4] used his models in the prediction of oilfields production decline rate types. Here Arps pointed out that there are 3-main types of production decline rate power constants (n). These are the constant or exponential decline rate (where n = 0), hyperbolic decline rate (where $0 < n < 1.0$) and harmonic decline rate (where n = 1.0). He plotted production data against time in a semi-log paper and found out that it gives a

straight line graph which could be extrapolated to estimate the oilfield reserves. This was possible, because the drop in production per unit time was a constant fraction of the hydrocarbon production rate.

$$a = -\frac{dq}{q dt} = \text{constant} \quad 2.2$$

In the hyperbolic decline rate, he (Arps) found out that the decrease in production per unit time as a fraction of the production rate is proportional to a fractional power. The coefficient of his fraction decline when

$$0 < n < 1.0 \text{ was given as: } N_p = \frac{q_i}{a(1+n)} (q_i^{1-n} - q^{1-n}) \text{ where } q = \frac{q_i}{(1+nat)^{\frac{1}{n}}}. \text{ The coefficient of the decline}$$

$$\text{rate for harmonic decline is unity (n = 1), so the equations become. } q = \frac{q_i}{(1+nat)} \text{ and } N_p = \frac{q_i}{a} \ln \frac{q_i}{q}.$$

EDWARDSON, ET AL (1962)^[5] provided the mathematical equation for cumulative hydrocarbons values estimation using dimensionless terms: When $t_D > 200$, $Q_D = \frac{-4.23 t_D^{0.5} + 2.026 t_D}{\ln t_D}$ and when

$$t_D < 200, Q_D = \frac{1.12838 t_D^{0.5} + 1.19328 t_D^{\frac{1}{2}} + 0.27 t_D^{\frac{1}{5}} + 0.086 t_D^{\frac{2}{5}}}{1 + 0.62 t_D^{0.5} + 0.041301 t_D^{\frac{1}{2}}} \quad 2.3$$

BRUNS, (1986)^[6] tried, using fractions as $\frac{1}{2}$, $\frac{5}{8}$ and $\frac{3}{4}$ in his dimensionless time-function and found out that using $\frac{1}{2}$ reduces the discontinuity between the transient streams and hyperbolic streams.

BAILEY, (1982)^[7] investigation showed that in some fractured gas wells the rate declined value ‘b’ is greater than unity and sometimes as high as 3.5.

FETKOVITCH, (1984)^[8], concluded that in commingled layered reservoirs the values of ‘n’ lies between 0.5 and 1.0. In such a case decline analysis should be initialized from the start of the decline rate. He added that it is possible under certain production and scenarios that initially the rate does not decline. **Fetkovitch** designed an advanced decline curves analysis approach, which has been applicable for changes in pressure or drainage. His approach was similar to pressure testing. $\frac{q}{q_i}$ VS at or q_{Dd} VS t_{Dd} He also

used different values of ‘n’, in Arps equations and plotted out curves. From these curves Fetkovitch concluded that Arps’ equations are only suitable for rate-time depletion data, but in transient time data will result in incorrect forecasts. In the full size type curves by Fetkovitch field data were plotted on a tracer paper, which are the same as log-log paper scale as the full-size types curves. The best fit in bbl/unit time would be chosen. A match can be used to obtain values of q_i & q for actual data. These data are then used for appropriate equations to be used in the analysis of the rate-time as well as cumulative hydrocarbons production (N_p or G_p).

BLASINGAME, ET AL (1989)^[9] introduced the concept of integral type curves in the well testing fields. They developed type curves which showed the analysis of transient stems along side with the analytical harmonic decline, but with the rest of the empirical hyperbolic stems absent. Blasingame’s hydrocarbons production decline techniques are not limited to constant bottom hole flowing pressure like those in Arps and Fetkovitch. Their hydrocarbons production decline techniques account for variations in bottom hole flowing pressure in the transient regime. In addition their analysis can work fine in the changing values of reservoir PVT properties with the changing reservoir pressure for both oil and gas. They also stated that, if a mechanism maintains the reservoir pressure, the production rate would remain fairly constant. This means that at constant reservoir pressure the decline tends to zero. This is common in pressure maintenance systems, such as gas & water injections, active-water drive, and gas-cap expansion drive, where the hydrocarbons are saturated. Small reservoir pressure decline leads to high production driving force with a corresponding small production decline rate. In this case the decline rate constant is theoretically greater than unity ($n > 1$). Much later when the oil column thins, the production rate would decline exponentially with $n = 0$ and the hydrocarbons production is replaced by water. Advantages in their work were the development

of oil and gas production decline method that uses superposition time function that only requires one depletion stem for type curves matching, one of the importance of his method was the type curves used for matching, were identical to those used for Fetkovitch decline analysis without the empirical depletion streams. When the type curves are plotted using Blasingame's superposition time function the analytical exponential stem of Fetkovitch's type curves becomes harmonic. The significance of this is that if the inverse of this flowing pressure is plotted against time, pseudo steady state depletion at constant flow rate follows a harmonic decline. In effect it allows depletion at a constant pressure to appear as pseudo steady state depletion at constant rate, provided that the rate and pressure decline monotonically.

ECONOMIDES, ET AL (1994)^[10], considered an oil well drilled in a volumetric oil reservoir where they assumed that the wells production rate starts to decline when a critical (lowest permissible) bottom hole pressure (BHP) is reduced. Under the pseudo-steady-state flow condition the production rate at a given decline time (t) was expressed mathematically as:

$$q = \frac{k h (P_t - P_{wf})}{141.2 B_o \mu \ln\left(\frac{0.472 r_e}{r_w}\right) + S} \quad \text{and} \quad N_p = \int_0^t \frac{k h (P_t - P_{wf}) dt}{141.2 B_o \mu \ln\left(\frac{0.472 r_e}{r_w}\right) + S} : N_p = \frac{C_t N_i}{B_o} (P_0 - P_t) \quad 2.4$$

Where: P_t = Average reservoir pressure at decline time, t and P_{wf} = Critical BHP during production decline. N_p = Cumulative production of the well after the decline time (t), C_t = Total reservoir compressibility, N_i = Initial oil in place in the well drainage area and P_0 = Average reservoir pressure at decline time zero.

RAMSAY AND GUERRERO, (2002)^[11], Study also included relative decline rate and they indicated in their work that about 40% of leases have $b > 0.5$ and commingled layered reservoirs fall between $0.5 < b < 1.0$.

KING-HUBBERT AND ROBERTSON, (2004)^[12], suggested in their work "Modified Hyperbolic Decline" that at some point in time the hyperbolic decline is converted into an exponential decline. They extrapolated hyperbolic decline over long periods of time and found out that it frequently results in unrealistically high pressure. To avoid this problem, they made their suggestion. They assumed that for a particular example, the decline rate (D) starts at 30% of flow and declines in a hyperbolic manner. When it reaches a specified value say 10% of the hyperbolic decline it converted to an exponential decline. The error here is that exponential decline rate of 10% would be considered in the forecast. Fig 2.4 shows the graphical representation of their work:

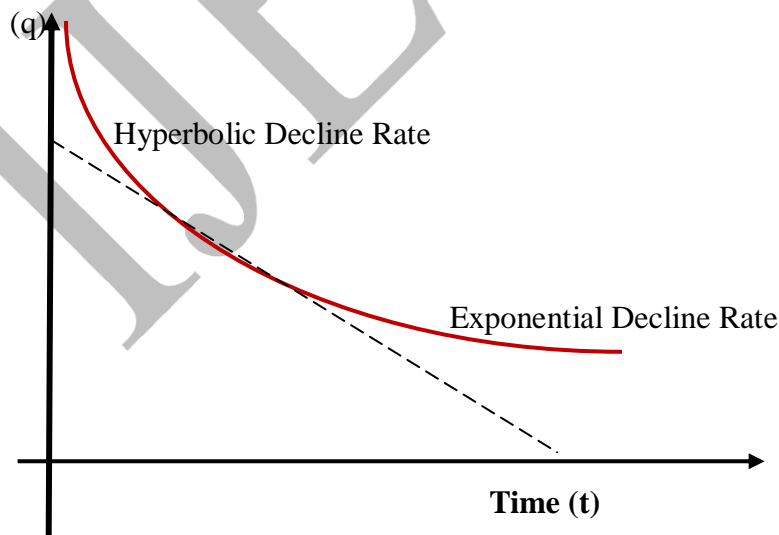


Fig 2.4 Conversion Hyperbolic into Exponential Decline Trend

MATHEMATICALLY:

$$q = q_i \frac{[(1-B)^b e^{-Dt}]}{1-B e^{(-Dt)^b}} \quad \text{and} \quad Q = q_i \frac{1-B}{BD} \left[1 - \frac{1}{1-B e^{(-Dt)^{b-1}}} \right] \tag{2.5}$$

When $b = 1$: $Q = q_i \frac{1-B}{BD} \ln \left[1 - \frac{1-B e^{-Dt}}{1-B} \right]$ Or $D = \frac{D_i}{q_i^{b-1} (1 + b D_i t)^{\frac{1}{b}}}$ and $q = q_i (1 + b D_i t)^{-\frac{1}{b}}$

AMINI, ET AL, (2007)^[13], reservoir model used elliptical flow to govern flow regime in a low permeability gas reservoir with elliptical outer binding. He described these cases as one production from an elliptical wellbore, elliptical fracture or a circular wellbore in an anisotropic reservoir system, which can be considered to be an elliptical inner boundary. They stated that an elliptical reservoir surrounded by an elliptic aquifer is an elliptical outer boundary. They also stated that the reservoir is assumed to be a single-layer system that is isotropic, horizontal and uniform thickness and constant flow rate. Mathematically:

$$q_D = \frac{141.2 B \mu q}{K h \Delta P} \quad \text{and} \quad K = \frac{141.2 B \mu}{h} \left[\frac{q / \Delta P}{q_D} \right] \tag{2.6}$$

AGARWAL AND GARDNER, (2008)^[14], presented new decline type curves for analyzing production data. Their method builds on Fetkovitch’s and Palacio-Blasigame’s ideas. They utilized the concept of the equivalence between constant rate and constant pressure solution. They also presented new type curves with dimensionless variables based on the conventional well-test definition as in Fetkovitch and Blasigame. They equally included primary and semi-log pressure derivatives plots (decline analysis inverse formant). They as well presented rate versus cumulative and cumulative versus time plots. Rate – cumulative Production analysis mathematically: $Q_{DA} = \frac{t_{DA}}{P_D} = q_D t_{DA}$ and $q_D = \frac{141.2 q B \mu}{K h (P_t - P_{wf})}$ and they explained the importance of water influx in gas reservoir. They observed that an appreciable water influx in a gas reservoir acts as pressure maintenance naturally delaying the decline initiation. The benefit is that much of the hydrocarbons are produced. The disadvantage is that such a reservoir is difficult to model, due to less knowledge of the aquifer behavior and life span.

ILK, ET AL (2008)^[15], presented the “Power - Law” decline method which uses a different functional form of D-Parameter given by: $D = D_\infty + D_1 t^{-(1-n)}$

2.7

D is approximated by a decaying power-law function from transient and through transition flow and exhibits a near constant behavior (*ie* D_∞) at very large time. This is contrast to hyperbolic rate decline that leads to a constant behavior at early time and becomes a unit slope power law decaying function at larger times. The advantage of their mathematical equation is that it is flexible enough to cover the transient, transition and boundary dominated flow and to large time reduces to an exponential decline ($D = D_\infty$). They then combined their equation with Arps’ equation as: $\frac{1}{D} = \frac{q}{dq/dt} + D = D_\infty + D_1 t^{-(1-n)}$, Solving

gives $q = q_i e^{[-D_\infty t - \frac{D_1}{n} t^n]}$

2.8

When, $D_1 =$ Decline constant, $t \rightarrow \infty$, $n =$ Time exponent and $q_i =$ Rate intercept at $t = 0$. The difference between their q_i and q_i in Arps decline models is because it refers to rate at the onset of stabilized flow, while q_i in Arps decline models refers to flow rate at early stage of a well.

OBAH, ET AL (2012)^[16] used a dynamic simulator and generated a 3D generic grid model with varying oil column thickness, gas-cap and aquifer size. Their based grid was 10 x 10 grid block in the x and y directions. The model geometry was fixed at 600ft x 600ft in the x and y directions, while the z-direction was varied based on the oil rim thickness. They obtained 3-production forecast models for oil rim reservoirs, using

Monte Carlo Simulation approach and generated a probabilistic range of forecasts for decision making in the Niger Delta, Nigeria for 30 years. They found out that oil recovery varies from 3.98 – 37.3MMstb over the 30years prediction. They concluded that horizontal wells are better option for developing reservoirs with oil rim as to conventional wells. They also added that oil recovery is strongly dependent on the oil rim thickness, relative gas-cap size (in-factor), permeability, viscosity and aquifer strength. Their mathematical equation was:

$$N_{p(t)} = \frac{q^*}{D} (e^{-D/t_p} - e^{-Dt}) + q_i t_p$$

2.9

REVIEWED EVALUATION AND RESEARCH PROPOSAL:

Evaluating the early researchers’ works, it is observed that the whole work is based on identifying exponential, hyperbolic or harmonic decline. They used semi-log fit or cross-match that an exact fit of data was not easily possible. The principal challenges were to improve reserves estimation errors, projecting future reserves production and time required for reserves recovery. The attempt to estimate reserves initially in place and the accuracy in DFCA has not been properly delineated. The gap I intent to fill is to improve reserves (N_p and N) estimation accuracy from 60/67% to 90/99%, reduce the time used in simulation, substituting the exponential, hyperbolic and harmonic decline constants with projectile and parabolic flow decline trends. This is because projectile and parabolic flows make it easy to achieve rate decline trends constant through flow order which had been difficult to achieve.

MATERIALS AND METHODS:

MATERIALS

The materials used for this work were collected from DPR, NNPC namely, daily operation logging data of oil and gas wells located in the Niger Delta areas. The wells covering Exploration (wildcat) wells, Appraisal (out-step) wells and Production (exploration development) wells. The main data were early to abandonment stages rates. The first set of data were specifically from the exploration, appraisal and production wells, because those wells could define early-stage to the actual production data records, while the second set of data were from the tanks-farms yearly production records (surface facilities) of the same Niger Delta formation oil wells. These were used mainly for validating the input data. Table 3.1 shows field data of gas reserves production for $22\frac{1}{2}$ years. Table 3.2 shows oilfield production data for 10years. Table 3.3 shows an oilfield production test data in one month (Sept. 1996). Table 3.4 shows an oilfield production test data in one year (1996): Well – 21A, and Table 3.5 shows an oilfield production test data in one year (1999): Well – 21B.

Table 3.1: Field Data for Gas Production in $22\frac{1}{2}$ Years

Date	Time, t (yr)	Rate q, MM scf/d
1977	0	0
1978	1	50
1979	2	100
1980	3	100
1981	4	100
1982	5	100
1983	6	100
1984	7	100
1985	8	100
1986	9	100
1987	10	100
1988	11	100
1989	12	100

1990	13	100
1991	14	100
1992	14.45	100
1993	15	89.60
1993	16	73.34
1994	17	60.05
1995	18	49.16
1996	19	40.25
1997	20	32.96
1998	21	26.98
1999	22	22.09
1999	22.5	20.00

Table 3.2: Delta State South Oilfield, March, 1968 to March, 1978

Date	Time, t (yr)	Pressure (Psi)	Rate, q (Stb/d)	Rate, q M (stb/yr)	Cumulative N_p , (M Stb)	B_o (rb/stb)
1968	0	4180	0	0	0	1.308
1969	1	4140	5498	2008.1	2008.1	1.301
1970	2	4119	6125	2237.1	4245.2	1.298
1971	3	4070	5885	2149.5	6394.7	1.297
1972	4	4032	6115	2233.5	8628.2	1.293
1973	5	3998	5640	2060.0	10688.2	1.290
1974	6	3960	4750	1735.0	12423.2	1.289
1975	7	3928	4500	1644.0	14067.2	1.285
1976	8	3930	2900	1059.0	15126.2	1.286
1977	9	3950	2345	856.5	15982.7	1.289
1978	10	398	1830	668.4	16651.1	1.299

Table 3.3 Oilfield Production Test Data in One Month (Sept. 1996)

Time, t (yr)	0.0	0.0831				
Rate, M stb/d	100	96.00				

Table 3.4 Oilfield Production Test Data in One Year (1996): Well – 21A

Time, t (yr)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Rate, M stb	96.3	92.9	89.80	86.80	84.00	81.40	79.00	76.70	74.50	72.50

Table 3.5 Oilfield Production Test Data in One Year (1999): Well – 21B

Time, t (yr)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Rate, M stb/d	95.63	92.8	89.50	86.40	83.50	80.70	78.10	75.50	73.20	70.90

RESEARCH METHODOLOGY:

Raw data for the analysis were collated or grouped into three main dynamic characterizations. Initials to abandonment rates of production, Initials to a given period rates of production and Short period production rates history.

EVALUATION MODELS – I: [GOVERNING MODELS]

Initial rates to abandonments were plotted against time to generate governing evaluation curves. The curves were used to obtain rates decline constant, “b”, the decline constant, “b” were used to predict yearly rates, the yearly rates were used to build evaluation models and the models were then used to estimate reserves

[N_p & N].

EVALUATION MODELS – II:

Initial rates to given periods of production were analyzed for decline constant, “b”, the decline constant, “b” was used to predict yearly rates, the yearly rates were used to generate evaluation curves to the given periods of production & extrapolated the curves to abandonment, the extrapolated curves were used to build evaluation models and the models were then used to estimate reserves [N_p & N].

Short periods production rates were equally analyzed for declined trends and constant, “b”, the declined constant, “b” was used to predict yearly rates to abandonments (called generic data), the generic data were used to generate evaluation curves, the curves were used to build evaluation models and the models were then used to estimate reserves [N_p & N]. Figure 3.1 below shows a flowchart of the data collation and figure 3.2 shows the flowchart for quality evaluations and applications.

ANALYSIS Procedures: Data Type – I

Plotting the collated data on Table 3.1 generated a projectile curve, Figure 3.1 and plotting the data on Table 3.2 generated projected curve of Figure 3.2.

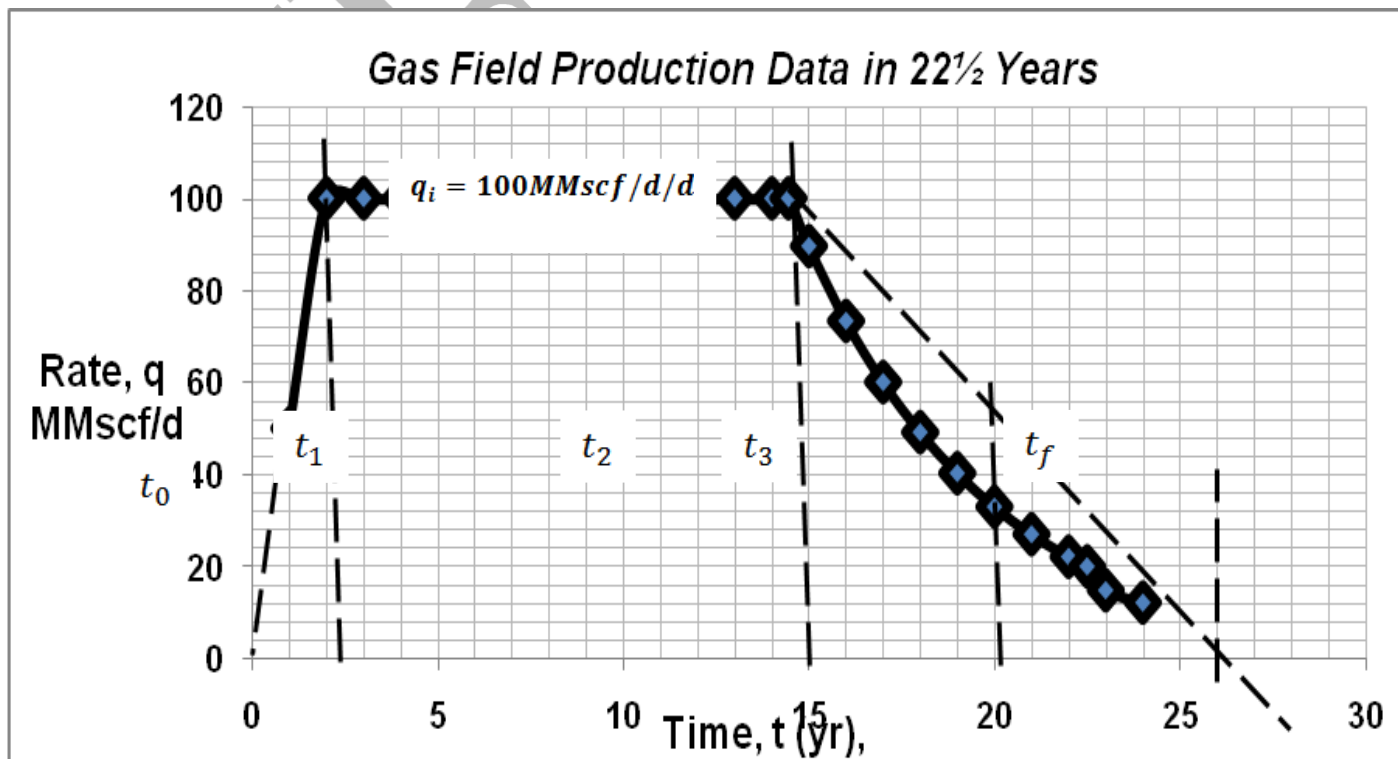


Fig 3.1 Projectile Decline Curve Using Table 3.1

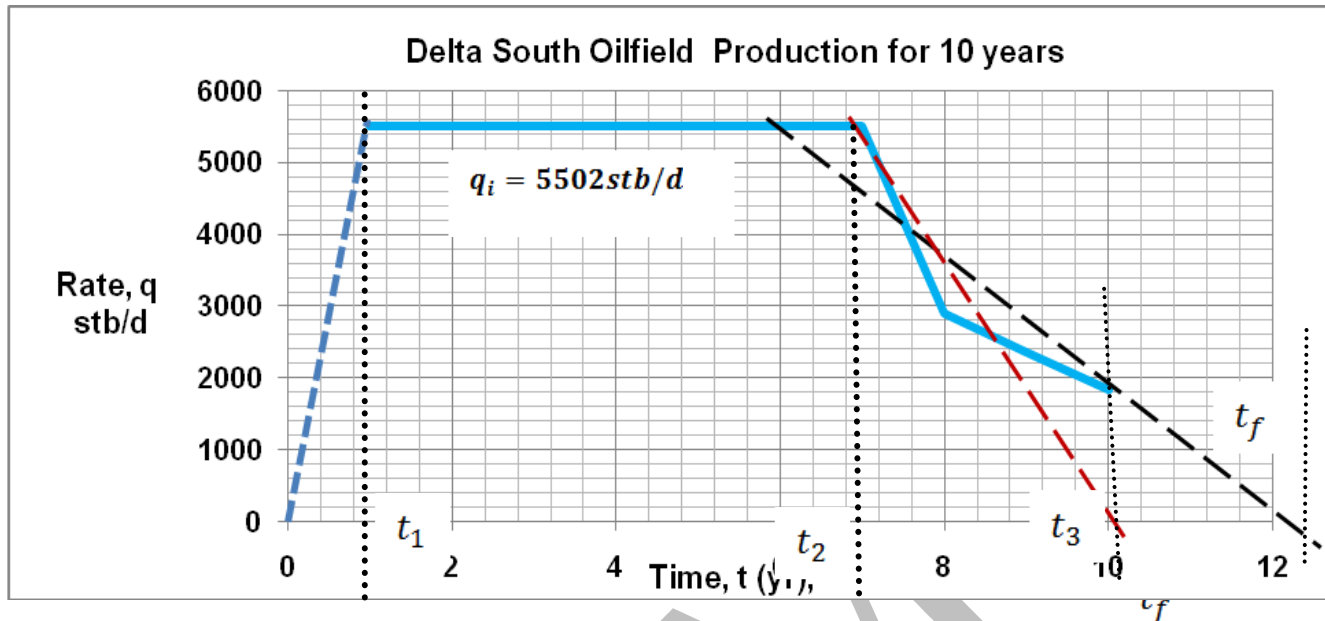


Fig 3.2 Projectile Decline Curve, using Table 3.2

POSTULATION OF THE PROJECTILE OIL FLOW MODELS:

In this section the principal method for postulating the evaluation models was the projectile dominated flow of the reserves. The projectile flow was found common in the depletion of saturated hydrocarbons reservoirs from the initial stage to an abandonment stage. Hydrocarbons reserves recovery values on table 3.1 were used in plotting the curves which were used to study the complete reserves recovery from the initial stage through the transient stage, steady stage, the decline stage to economic rate called abandonment rate (Figure 3.1) and yearly oil recovery data on table 3.2 were used to study complete oil recovery (Figure 3.2). The resulted curves in projectile shapes were used to build the models for studying the decline trends and projected to both given recovery periods for estimating the cumulative reserves and zero declined for estimating the reserves initially in place ($[N_p \& N]$).

CONSIDERATIONS POINTS:

An oilfield must contain a reserve initially in place (N), which reduces per unit time, due to hydrocarbons production operations. The flow rate (q) of oil stream production continues to change from time- t_0 to time- t_1 and from time- t_1 to time- t_2 and from time- t_2 to time- t_3 , (Figure 3.3), so that time- t_f could be extrapolated to the initial reserves values. The hydrocarbons production (N_p) per unit time declined from the initial value to minimum ($\frac{dq}{dt} = -bq^n$). The constant of proportionality is $-b$. The quantity of the reserves remaining in the reservoir is N_f . **Construction:** Pt-B was joined to pt-E giving the trapezium ABEO and pt-B to pt-D giving the trapezium ABDO respectively. Eqn3.1 is the general material balance equation (MBE).

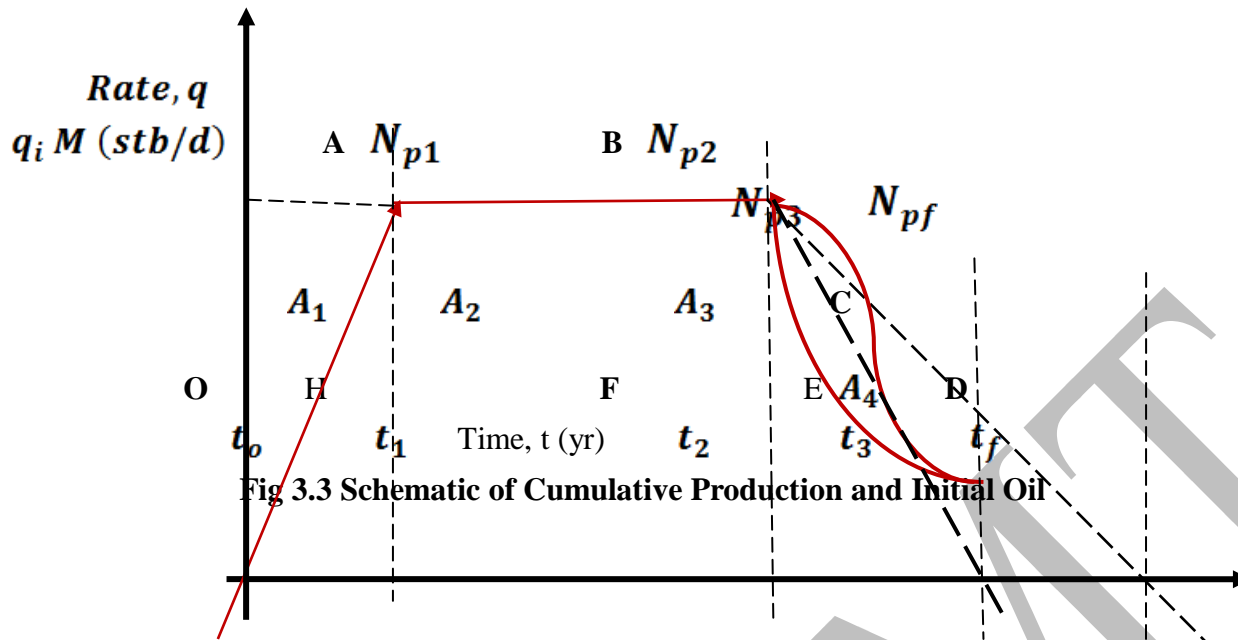


Fig 3.3 Schematic of Cumulative Production and Initial Oil

EVALUATION MODEL – 1: THE PROJECTILE OIL FLOW

$$\left[\begin{matrix} \text{Actual Reserves} \\ \text{Produced in a} \\ \text{Given Time} \end{matrix} \right] = \left[\begin{matrix} \text{Actual Reserves} \\ \text{Initially in Place} \end{matrix} \right] - \left[\begin{matrix} \text{Actual Reserves} \\ \text{remaining in Place} \end{matrix} \right]$$

$$N_p = N - N_f \tag{3.1}$$

Using Figure 3.3, the actual reserves produced in a given time and initially in place are expanded as:

[Oil Produced] = [Area of the Trapezium ABEO]

$$[\text{Oil Produced}] = \frac{1}{2} [\text{Sum of the Parallel Sides}] * [\text{Height}]$$

$$N_p = \frac{q_i}{2} [(t_2 - t_1) + (t_3 - t_0)] \text{ Or } [A_1 + A_2 + A_3] \tag{3.2}$$

$A_1 = \frac{q_i}{2} [t_1 - t_0] = \text{Area of } \Delta AHO$, $A_2 = q_i [t_2 - t_1] = \text{Area of the rectangle } ABFH$ and

$A_3 = \frac{q_i}{2} [t_3 - t_2] = \text{Area of } \Delta BEF$. Substituting A_1, A_2 and A_3 , gives Eqn3.3. If you so desired,

using equation of the curve part ($A_3 = N_{p3} = \frac{q_i}{b} (1 - e^{-bt})$) in Figure 3.3, gives the Oil recovered in Decline rate Stage. This model derived from the first principle below.

$$N_p = \frac{q_i}{2} [(t_2 - t_1) + (t_3 - t_0)] \tag{3.3}$$

Yearly Hydrocarbons Production Projection

The general equation for natural production of an oilfield reserves is the product of the rate-constant and the actual rate raised to power-n. This is given mathematically by Eqn3.4:

$$\left[\begin{matrix} \text{Actual Change} \\ \text{in Production} \\ \text{Rate with Time} \end{matrix} \right] = \left[\begin{matrix} \text{A Decline Rate} \\ \text{Constant} \end{matrix} \right] \left[\begin{matrix} \text{Actual Rate in} \\ n - \text{order} \end{matrix} \right]$$

$$\frac{dq}{dt} = -bq^n \tag{3.4}$$

Using the curve in Figure 3.3 and integrating Eqn3.4, gives the governing equation, Eqn3.6. The governing equation, Eqn3.6 is used to postulate actual yearly oil production rate (q) by removing the log and rearranging giving Eqn3.7. To estimate the rate-constant (b), the governing equation, Eqn3.6 is rearranged to obtain Eqn3.7. Three main flow orders of decline rates were considered.

When n = 1: **1st Order Decline Rate Parabolic Flow**

$$\int_{q_i}^q \frac{dq}{q} = -b \int_0^t dt \tag{3.5}$$

$$\ln q - \ln q_i = -bt \tag{3.6}$$

$$q = q_i e^{-bt} \quad \text{and} \quad b = \frac{\ln(q_i/q)}{t - t_i} \tag{3.7}$$

CUMULATIVE RESERVES PRODUCTION MODELS:

The general equation for natural production of an oilfield reserves is the product of the hydrocarbons flow rate and the actual time elapsed. This is given mathematically in Eqn3.8. In Eqn3.7, $q = q_i e^{-bt}$, so substituting this q in Eqn3.8 gives Eqn3.9. Solving Eqn3.9 gives Eqn3.10, the governing evaluation models postulated for actual oil cumulative production.

$$\left[\begin{matrix} \text{Actual Cumulative} \\ \text{Hydrocarbons} \\ \text{Produced With Time} \end{matrix} \right] = \left[\begin{matrix} \text{Actual Production} \\ \text{Rate Per Unit Time} \end{matrix} \right] \left[\begin{matrix} \text{Actual Time} \\ \text{That Elapsed} \end{matrix} \right]$$

$$N_p = \int q dt \tag{3.8}$$

$$N_p = \int_0^t q_i e^{-bt} dt \tag{3.9}$$

$$N_p = \frac{q_i}{b} [1 - e^{-bt}] \quad \text{For oil systems}$$

3.10

This implies that the projected hydrocarbons production is: $N_{p3} = A_3 = \frac{q_i}{b} (1 - e^{-bt})$

Oil-Reserves Initially in Place (N) Model Postulation for Projectile Oil Flow

$$\left[\begin{matrix} \text{Actual Oil Reserves} \\ \text{Initially in Place} \end{matrix} \right] = \left[\begin{matrix} \text{Area of the} \\ \text{Trapezium ABDO} \end{matrix} \right] \text{ Fig 3. 1}$$

$$\left[\text{Area ABDO} \right] = \frac{q_i}{2} [(t_2 - t_1) + (t_f - t_o)]$$

$$N = \frac{q_i}{2} [(t_2 - t_1) + (t_f - t_o)] \tag{3.11}$$

Equation 3.11 is the actual stock tank oil initially in place (N). This is very possible since hydrocarbons production is the product of the flow rate, q and time, t (N = q * t).

POSTULATION OF THE PARABOLIC OIL FLOW MODELS:

DATA TYPE – II AND TYPE III: CONSIDERATIONS:

An oilfield must contain a reserve initially in place (N), which reduces per unit time, during production operations. A reserve must decline right from initial stage during production in a parabolic dome-shape (Figure 3.4a) or single-apex shape (Figure 3.4b). The flow rate (q) of oil stream production continues to change from time, t_0 to time, t_1 , so that time, t_f could be estimated, by extending Pt-X to Pt-y at time- t_f . The actual change in a production rate per unit time is $dq \propto q^n dt$ and the constant of proportionality is $-b$ or it is the product of the decline rate constant, b and flow rate raised to power- n ($-bq^n$). The cumulative hydrocarbons production (N_p) per unit time would be reduced from the maximum at bubble point (transition state) value to minimum at a given time. The quantity of the reserves remaining in the reservoir is N_f at time t_f .

Parabolic Flow Types – 2, with short or no Transient and Transition Times

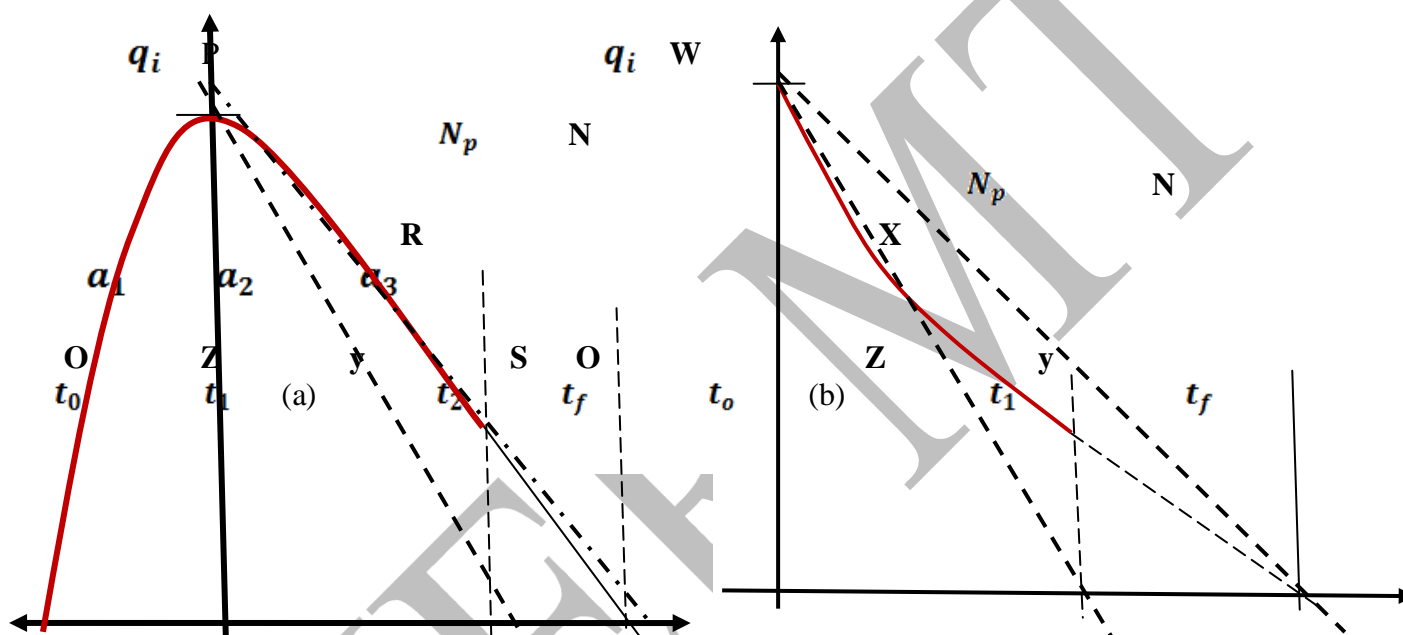


Fig 3.4 Schematic of Oil in Parabolic Flow Regime

EVALUATION MODEL – II

The dome shape of Fig 3.4a indicates a parabolic flow rate from lowest at point-P to a maximum point –P and declines to abandonment at point –R. The curve was extrapolated from point- R at t_1 to point –S at t_f , for estimation of oil-reserve initially in place by extension of curve-PR at point-R to S in time- t_f . In the case of Figure 3.4b the reservoir pressure is just slightly above the bubble point or at bubble point pressure. The implication of this case is that decline starts right from the early age of production at point–W to point-X. The curve was extrapolated from point- X to point–y, for estimation of oil initially in place. The early production data were projected to induced abandonment periods for estimating the cumulative hydrocarbons production and hydrocarbons reserves initially in place. The postulated models determinant confirmation equations were the projectile dominated fluid flow and the field recovery results. Table 3.1 shows the projectile dominated hydrocarbons (gas) production trend and table 3.2 shows the projectile dominated hydrocarbons (oil) production trend. The outstanding advantages of the decline stage models include: Prediction of the daily oil production rate and cumulative recovery in a given period. This enables the

operator to equally predict the abandonment period and the cumulative recovery value. Good prediction of the reserves in place when the decline rate stage is converted to a projectile dominated flow stream.

HYDROCARBONS PRODUCTION RATE DECLINE CONSTANT AND RATES MODELS:

Basically 3 types of rate decline trends were used 1st order equation where $n = 1$, 2nd order equation where $n = 2$ and fraction order equation where $n < 1$ or $n < 2$. The general equation for natural production of an oilfield reserves is the product of the rate-constant and the actual rate raised to power- n . This is given by parabolic flow regime (Eqn3.12): Using the curve in Figure 3.4a and integrating Eqn3.12, the Governing Evaluation model was postulated. The governing equation was used to obtain hydrocarbons production rate, q and the rate-constant (b). To obtain the rate, q , remove the log and rearranging gives $q = q_i e^{-bt}$. To estimate the rate constant (b), the governing equation is applied at point-A, point-B and point-C of Figure 3.5, generating 3 equations and solving simultaneously each pair for “ b ” Egn3.15 as follows:

$$\left[\begin{matrix} \text{Actual Change} \\ \text{in Production} \\ \text{Rate with Time} \end{matrix} \right] = \left[\begin{matrix} \text{A Decline Rate} \\ \text{Constant} \end{matrix} \right] \left[\begin{matrix} \text{Actual Rate in} \\ n - \text{order} \end{matrix} \right]$$

$$\frac{dq}{dt} = -bq^n \tag{3.12}$$

When $n = 1$: 1st Order Decline Rate Parabolic Flow

$$\int_{q_i}^q \frac{dq}{q} = -b \int_0^t dt \quad \text{or} \quad \ln q - \ln q_i = -bt \quad (\text{The Governing Equation})$$

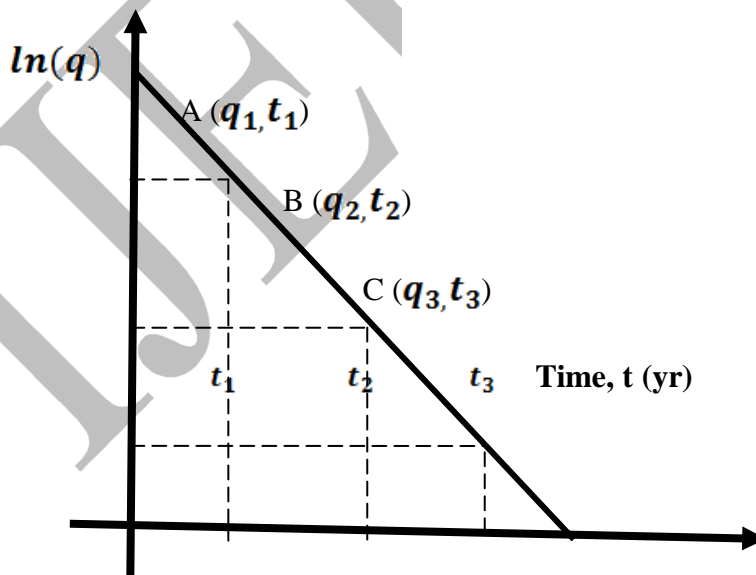


Fig 3.5 Parabolic Rate Decline Plot

$$\ln q_1 - \ln q_i = -bt_1 \tag{3.13}$$

$$- (\ln q_2 - \ln q_i = -bt_2) \tag{3.14}$$

$$(3.13) - (3.14) \quad \ln \frac{q_1}{q_2} = b(t_2 - t_1) \quad \text{or} \quad b = \frac{\ln(q_1/q_2)}{t_2 - t_1} \quad \text{and} \quad q = q_i e^{-bt} \tag{3.15}$$

If $b_1 = b_2 = b_3 = \dots = b_n$ it implies uniform decline and $n = 1$, so the equation $b = \frac{\ln(q_1/q_2)}{t_2 - t_1}$, was used to project the flow rate, q for a give time, t . Using $q = q_i e^{-bt}$: q_1 at t_1 , q_2 at t_2 , q_3 at t_3 , \dots q_n at t_n

When $n = 2$: 2nd Order Decline Rate Parabolic Flow

Solving Eqn3.16 and Multiplying LHS by q_i and rearrange gives Equ3.17, the governing equation: To estimate the rate constant, “b” the governing equation is similarly applied at point-A, point-B and point-C of Figure 3.5, generating 3 equations and simultaneously each pair is solved for “b” or just re-arranged making b the subject of the formular ($q = q_i - bqt$). Plotting q vs t , the slope is $-bq$ and intercept is q_i .

$$\int_{q_i}^q \frac{dq}{q^2} = -b \int_0^t dt \text{ or } \frac{1}{q_i} - \frac{1}{q} = -bt \tag{3.16}$$

$$q = \frac{q_i}{(1 + bt)} \text{ or } q = q_i - bqt \text{ (2nd order flow governing equation)} \tag{3.17}$$

$$\begin{aligned} q_1 &= q_i - bq_1 t_1 \\ -(q_2 &= q_i - bq_2 t_2) \end{aligned} \tag{3.18}$$

3.19

$$(3.19) - (3.20) \quad q_1 - q_2 = b(q_2 t_2 - q_1 t_1) \text{ or } b = \frac{q_1 - q_2}{q_2 t_2 - q_1 t_1} \text{ or } b = \frac{q_i - q}{q \Delta t} \tag{3.20}$$

If $b_1 = b_2 = b_3 = \dots = b_n$, indicating a uniform decline rate when, $n = 2$, so the equation, $b = \frac{q_1 - q_2}{q_2 t_2 - q_1 t_1}$ or $\frac{q_i - q}{q t}$, was used to project the flow rate, q for a given time, t . That is q_1 at t_1 , q_2 at t_2 , q_3 at t_3 , \dots q_n at t_n , when $n = 2$.

When $n < 1$ or $1 < n < 2$: Fraction-Order

The value of, $b_1 \neq b_2 \neq b_3 \neq \dots \neq b_n$, it indicates non-uniform decline rate. In this case an average production rate decline was used to estimate decline constant, b using each point, in the projected flow rate, q within the given time, t . This means b_1 at t_1 is calculated and used for N_{p1} , b_2 at t_2 is equally calculated and used for N_{p2} , b_3 at t_3 is also calculated and used for N_{p3} , and so on to b_n at t_n for N_{pn} ,

$$q = \frac{q_i}{1 + bt} \quad (i = 0, 1, 2, 3, \dots, n) \text{ and } b \approx \sum_i^n \left[\frac{q_i - q_{i+1}}{q_{i+1} t_{i+1}} \right] \text{ Meaning:} \tag{3.21}$$

$$b \approx \frac{1}{n} \left[\frac{q_i - q_1}{q_1 t_1} + \frac{q_i - q_2}{q_2 t_2} + \frac{q_i - q_3}{q_3 t_3} + \dots + \frac{q_i - q_n}{q_n t_n} \right] \tag{3.22}$$

CUMULATIVE OIL-PRODUCED (N_p) AND RESERVES INITIALLY IN PLACE (N) MODELS:

In this case the reservoir started by building up the internal energy for some time from time, t_o to time, t_1 in figure 3.4a, because the reservoir was fairly saturated, so failed to attain boundary dominated flow at initial state. Instead it built-up from the initial stage to the transient and transition stage at point-P, but the flow period was too short. To this effects steady state flow (called the plateau) was not observed in the curve at time, t_1 instead rate decline state set in with short transition state, from time, t_1 to time, t_2 , Which covered cumulative oil recovery value (N_p , Mstb). After this the rate decline state continued from time, t_2 to time, t_f covering the total or Oil-Reserves initially in place value (N, Mstb). Any recovery from time, t_2 to time, t_f

covers the hydrocarbons supposed to be the residual oil of that reservoir. The complete depletion of the hydrocarbons in that reservoir from time- t_0 to **time- t_2** was cumulative oil recovery value and from time, t_0 to **time, t_f** was called hydrocarbons initially in place. The equation of the area of that shape (parabola) was the value of the hydrocarbons initially in place (Fig 3.4a/b). This is only obtainable in theory for reserves estimation, so it is an extrapolated value.

Parabolic Flow Type–1: Short Transient and Transition Times (Figure 3.4a)

Hydrocarbons Production per Unit Time (Mstb/yr) Model

$$\left[\begin{matrix} \text{Total Hydrocarbons} \\ \text{Production per Time} \end{matrix} \right] = \left[\begin{matrix} \text{Area of} \\ \text{Curve, } a_1 \end{matrix} \right] + \left[\begin{matrix} \text{Area of} \\ \text{Curv, } a_2 \end{matrix} \right]$$

$$N_p = \frac{q_i}{2} [(t_1 - t_0) + (t_2 - t_1)] \quad \text{For Oil} \quad 3.23$$

Hydrocarbons Initially in Place, Mstb (Figure 3.11) Models

$$\left[\begin{matrix} \text{Total Hydrocarbons} \\ \text{in Place initially} \end{matrix} \right] = \left[\begin{matrix} \text{Area of} \\ \text{Curve, } a_1 \end{matrix} \right] + \left[\begin{matrix} \text{Area of} \\ \text{Curv, } a_2 + a_3 \end{matrix} \right]$$

$$N = \frac{q_i}{2} [(t_1 - t_0) + (t_f - t_1)] \quad \text{For Oil} \quad 3.24$$

PARABOLIC FLOW TYPE–2: NO TRANSIENT OR TRANSITION TIMES (FIGURE 3.4B)

CUMULATIVE RESERVES PRODUCTION EVALUATION MODELS

Fig 3.4b shows that the oilfield reservoir was on its peak value, but the boundary conditions were observed right from the early stage. That was so because the reservoir was not externally supported. The challenge here is that it may be difficult to deplete the reservoir completely, due to multiphase flow effects..

$$\left[\begin{matrix} \text{Total Hydrocarbons} \\ \text{Production per Time} \end{matrix} \right] = \left[\begin{matrix} \text{Area of} \\ \text{Curve, } a_1 \end{matrix} \right] + \left[\begin{matrix} \text{Area of} \\ \text{Curv, } a_2 \end{matrix} \right]$$

$$N_p = \frac{q_i}{2} [t_1 - t_0] \quad \text{or} \quad N_p = \frac{q_i}{b} [1 - e^{-bt}]: \quad \text{Ref: Eqn3.10)} \quad 3.25$$

Reserves Initially in Place (stb) Evaluation Models (Figure 3.4b)

$$\left[\begin{matrix} \text{Total Hydrocarbons} \\ \text{in Place initially} \end{matrix} \right] = \left[\begin{matrix} \text{Area of} \\ \text{Curve, } a_1 \end{matrix} \right] + \left[\begin{matrix} \text{Area of} \\ \text{Curv, } a_2 + a_3 \end{matrix} \right]$$

$$N = \frac{q_i}{2} [t_f - t_0] \quad \text{For Oil} \quad 3.26$$

Application of the Model Equations Using Regional Data

Using the curve in Fig 3.1, $q_i = 100 \text{ MMscf/d}$, $t_o = 0$, $t_1 = 2 \text{ years}$, $t_2 = 14.45 \text{ yrs}$, $t_3 = 22.5 \text{ yrs}$ and $t_f = 25.8$ were estimated. Putting these values in Eqn3.7, the decline constant was obtained as: $b_{15} = \ln \frac{q_{15}}{q_{16}} =$

$$b_{16} = \ln \frac{q_{16}}{q_{17}} = b_{17} = \ln \frac{q_{17}}{q_{18}} = \dots = b_n = \frac{q_{n-1}}{q_n} = 0.2$$

, Using Eqn3.2, the cumulative hydrocarbons production (G_p) was obtained and in Eqn3.11 hydrocarbons initially in place (GIIP) was also obtained. These were comparable with Standing and Katz, (1942) MBE for volumetric hydrocarbons reservoir.

Ref: Tab 4.4: $G_p = 637.06 * 10^9 \text{ scf}$ and $GIIP = 699.7 * 10^9 \text{ scf}$.

$$G_p = \frac{365.25 * 100}{2} [(14.45 - 2.0) + (22.5 - 0)] = 638.274 \text{ MMM scf}$$

$$G = \frac{365.25 * 100}{2} [(14.45 - 2) + (25.8 - 0)] = 698.541 \text{ MMMscf}$$

The challenge was that the computer could not extend the curve scaled axis to abandonment stage ordinarily, so the projected values enabled that.

Similarly, extrapolation of the curve in Fig 3.2 from time t_3 to time, t_f . It was possible to estimate the hydrocarbons (Oil) initially in place. Data from the field records and solution from the curve (Figure 3.2) showed that, $q_i = 6000 \text{ stb}$, $t_o = 0$, $t_1 = 1 \text{ yr}$, $t_2 = 7 \text{ yrs}$, $t_3 = 10 \text{ yrs}$ and $t_f = 12.5 \text{ yrs}$. The decline constant, $b_1 \neq b_2 \neq b_3 \neq \dots \neq b_n$ was not uniform, Eqn3.22 was used to obtain ‘b’. Putting this values in the Eqn3.23, the cumulative oil production (N_p) was obtained and in the Eqn3.24, Oil initially in place (OIIP) was also obtained. The values were comparable with tank results (Table 4.4). The recovery factor was 83.26%.

$$N_p = \frac{365.25 * 5502}{2} [(7 - 1) + (10 - 0)] = 16,877 \text{ M stb} \quad N = \frac{365.25 * 6000}{2} [(7 - 1) + (12.5 - 0)] = 20,271 \text{ Mstb}$$

The challenge in this case was to curve-fit the plotted figure in order to extrapolate to the initial stage, so average value from zero to the 6th year was used as initial rate at the steady stage. The second decline rate trend was observed later when the reservoir pressure was enhanced by reservoir pressure maintenance by the Operator.

APPLICATION OF THE EVALUATION MODELS USING GENERIC DATA

The importance of generic or projected data is to project future field performance where we have short period of production data which are equally used to estimate oil initially in place. (i) In a production test Table 3.3, an oil-well flow rate declined from 100 Mstb/d to 96 Mstb/d in a month. The challenge was to predict the production rate, the cumulative reserves recovery in 5years and reserves initially in that reservoir.

SOLUTION - I

PREDICTION OF THE PRODUCTION RATE (ASSUMING: $n = 1$)

$q_i = 100 \text{ Mstb/d}$, $q = 96 \text{ Mstb/d}$, $t_o = 0$ and $t = \frac{1}{12} = 0.083 \text{ yr}$ Putting these values in eqn3.7 the decline rate constant (b) was obtained and in eqn3.7, $q_1 = q_i e^{-bt} = 100 e^{-0.4900 * 1} = 61.27$. The rate was tabulated on Table 3.6 and plotted against time, generated figure 3.8, which shows the curve for the projected rate.

Table 3.6: Yearly Projected Oil Production Rate from 1 Month Data

Date	Time T, (Yy)	Rate, q Mstb/d)	Rate, q M stb/yr))	Cumulative G_p , M Stb
1996	0	100.0	-	-
1997	1	61.26	28,875.14	28,875.14
1998	2	37.55	17,681.54	46,556.68
1999	3	23.00	10,842.62	57,399.30
2000	4	14.09	6,641.28	64,040.58
2001	5	8.63	4,302.40	68,342.98
Total Oil Production in five years: 68,572,895.59 M stb				

[Source: Generated from Table 3.3]

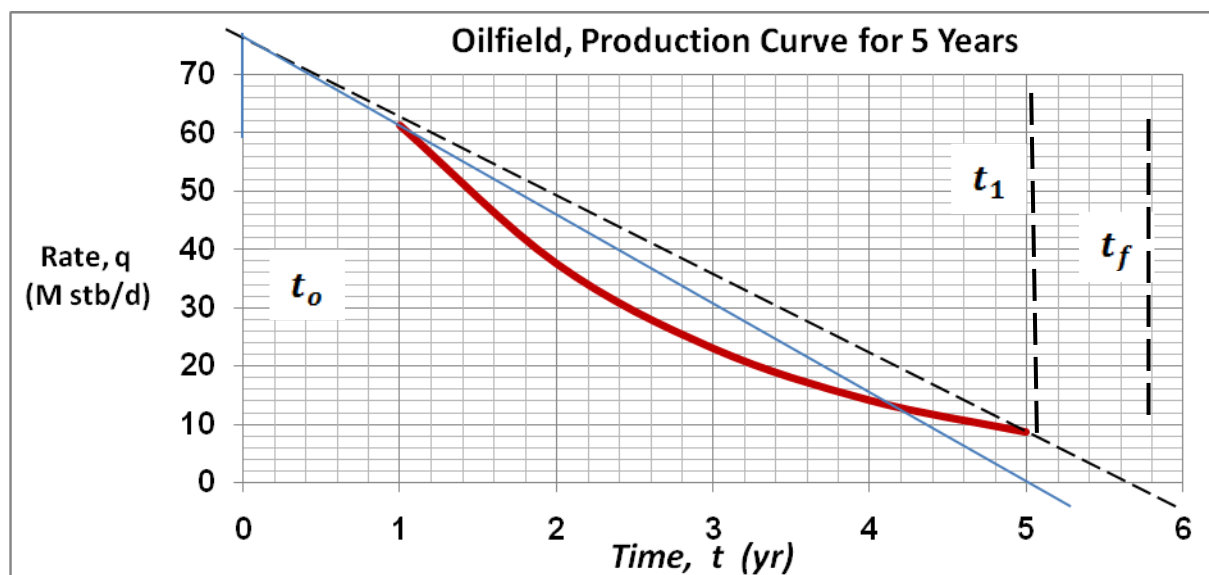


Fig 3.6: Curve Generated from the Projected Rate Data on Table 3.8

a. ESTIMATION OF THE HYDROCARBONS RECOVERY:

Solution from the curve showed that, $q_1 = 75.0stb$, $t_o = 0$, $t_1 = 5yrs$ and $t_f = 5.8yrs$. Putting these values in the model Eqn3.73, the cumulative oil production (N_p) was obtained as:

$$N_p = \frac{365.25 * 750}{2} [5 - 0] = 68,484.38M\ stb$$

b. ESTIMATION OF THE HYDROCARBONS INITIALLY IN PLACE:

The curve was extrapolated from time- t_1 to time- t_f and Oil-Reserve initially in place was estimated using the model Eqn3.7 as:

$$N = \frac{365.25 * 100}{2} [5.8 - 0] = 105,922.5Mstb$$

c. THE HYDROCARBONS RECOVERY FACTOR (E_R):

Recovery factor is the ratio of the cumulative oil production to the reserve initially in place. The challenge in this very short production history was to identify the decline trend. The only remedy was to produce a well

from initial rate to 1st, 2nd, 3rd, 4th or more decline rates to ascertain the production rate decline trend.

MATHEMATICALLY: $E_R = \frac{100 * N_p}{N} = \frac{100 * 68484.38}{105922.5} = 64.67\%$

Table 3.4 page 52 shows a production test of an oil-well that was produced for 1 year and the flow rate declined from 100Mstb/d to 70.9Mstb/d in a year. The challenge was to: predict the yearly production rate, cumulative oil production in 10 years and estimate oil-reserve initially in place and its recovery factor. Evaluation models postulated earlier were used and predicted the production rate in 10 years and equally estimated the cumulative oil recovery in 10 years. The projected rate values were used and generated a curve Figure 3.7. The curve generated was used in the confirmation of the evaluation models, which were used. Solution-II shows the estimated cumulative hydrocarbons produced in 10 years and the hydrocarbons initially in place.

SOLUTION - II

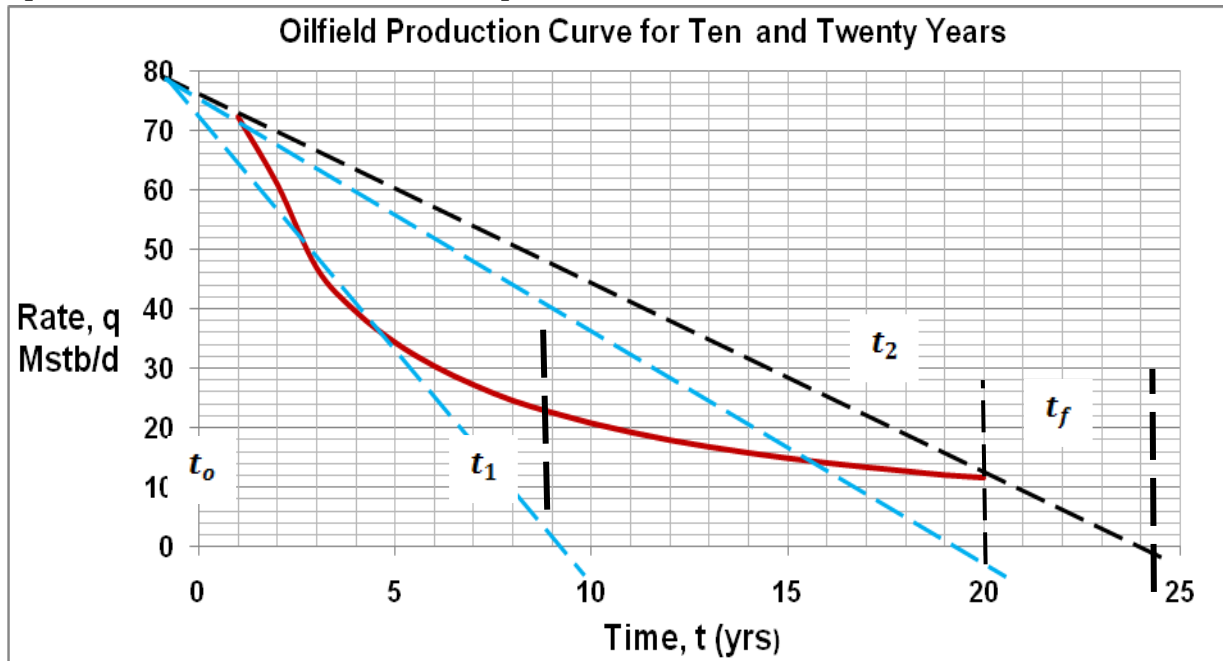
PREDICTION OF THE PRODUCTION RATE:

Using Table 3.4, $q_1 = 96.3 M stb/d$, $q_2 = 92.9 M stb/d$, and $q_3 = 89.8 M stb/d$ $t_1 = 0.1$, $t_2 = 0.2$ and $t_3 = 0.3yr$. Putting these values in Eqn3.20 the decline rate constant (b) was obtained. The decline rate exponent was a 2nd order decline trend (n = 2) as follows: $b = \frac{q_i - q}{q t} = \frac{100 - 92.9}{92.9 * 0.2} = 0.3821$ and substituting "b" in Eqn3.17, the yearly flow rate was also obtained as $q_1 = \frac{q_i}{1 + bt_1} = \frac{100}{1 + 0.3821 * 1} = 72.35$ and the results tabulated on Table 3.7. Figure 3.7 shows the curve generated from the projected rate data on Table 3.4.

Table 3.7: Yearly Oil Projected Production Rate Well – 21A

Date	Time T, (Yy)	Rate, q M stb/d	Rate, q MM stb/yr))	Cumulative N _p , M Stb
1996	0	-	-	-
1997	1	72.35	26,430.68	26,430.68
1998	2	56.68	20,702.37	47,133.05
1999	3	46.59	17,017.00	64,250.05
2000	4	39.55	14,445.63	78,555.68
2001	5	34.36	12,549.99	91,145.67
2002	6	30.37	11,092.64	102,238.31
2003	7	27.21	9,938.45	112,176.76
2004	8	24.55	8,966.89	121,143.65
2005	9	22.53	8,229.08	129,372.73
2006	10	20.74	7,575.29	137,000.10
Total Oil Production in ten years:			138,795 M stb	

[Source: Generated from Table 3.7]



a. ESTIMATION OF THE HYDROCARBONS RECOVERY:

Solution from the curve/graph showed that the rate, $q_1 = 76 \text{ stb/d}$, $t_0 = 0$, $t_1 = 10 \text{ yrs}$, $t_2 = 20 \text{ yrs}$ and $t_f = 24 \text{ yrs}$. Putting these values in the Eqn3.25 cumulative oil (N_p) recovery was estimated and in Eqn3.26 Oil initially in place (N) was estimated. as follows:

i. $N_p = \frac{365.25 \cdot 76}{2} [10 - 0] = 138,795 \text{ Mstb}$ (ii) $N_p = \frac{365.25 \cdot 20}{2} [20 - 10] = 36,525 \text{ Mstb}$

If the reservoir pressure was maintained early enough, say from ten years up to 20 years the total oil recovery would have been improved as shown below: $N_p = \frac{365.25 \cdot 76}{2} [20 - 0] = 277,590 \text{ M stb}$

b. ESTIMATION OF THE HYDROCARBONS INITIALLY IN PLACE:

Extrapolation of the curve from time t_1 to time, t_f I was able to estimate the hydrocarbons (Oil) initially in place using the Eqn3.26 as: $N = \frac{365.25 \cdot 100}{2} [24 - 0] = 438,300 \text{ Mstb}$

c. THE HYDROCARBONS RECOVERY FACTOR (E_R):

Recovery factor is the ratio of the cumulative hydrocarbon production to the hydrocarbon initially in place.

Mathematically: $E_R = \frac{100N_p}{N} = \frac{100 \cdot 138795000}{438300000} = 31.67\%$ and $E_R = \frac{100N_p}{N} = \frac{100 \cdot 277590000}{438300000} = 63.33\%$

If the reservoir pressure was maintained the recovery factor would have been improved as shown below:

$E_R = \frac{100N_p}{N} = \frac{100 \cdot 277590000}{438300000} = 63.33\%$

Table 3.5 Page 13 shows a production test of an oil-well which was produced for 1 year and the flow rate declined from 100 Mstb/d to 72.50 Mstb/d . The challenge was to predict the production rate in 10 years, estimate the cumulative hydrocarbons recovery in 10years and estimate the hydrocarbons initially in place. To match up the challenge the models postulated earlier were to predict the production rate for ten years and the corresponding hydrocarbons recovery. The result was tabulated on Table 3.8. The projected rate values were used to generate a curve, Figure 3.8. The curve generated was used in the confirmation of the

evaluation model. Solution-III shows the estimated cumulative hydrocarbons produced in 10 years and the hydrocarbons initially in place.

SOLUTION – III

PREDICTION OF THE PRODUCTION RATE ($1 > n < 2$)

Putting the values on Table 3.5 into Eqn3.22 the average decline rate was obtained. Substituting the average "b" in Eqn3.21, the yearly flow rate was also obtained and tabulated on Table 3.8. Figure 3.8 shows the curve generated from the projected rate/data on Table 3.8. Eqn3.25 was used to estimate cumulative oil recovery and Eqn3.26 to estimate Oil initially in place (N).

$$b \approx \frac{1}{n} \left[\frac{[q_i - q_1]}{q_1 t_1} + \frac{q_i - q_2}{q_2 t_2} + \frac{q_i - q_3}{q_3 t_3} + \dots + \frac{q_i - q_n}{q_n t_n} \right] \approx 0.4/yr$$

$$q_1 \approx \frac{q_i}{1 + bt_1} = \frac{100}{1 + 0.4 \cdot 1} \approx 71.00 \text{ M stb/d} \quad \text{and} \quad q_2 \approx \frac{q_i}{1 + bt_2} = \frac{71.00}{1 + 0.4 \cdot 1} \approx 55.56 \text{ M stb/}$$

Table 3.8: Yearly Projected Oil Production Rate from Well – 21B

Date	Time T, (Yy)	Rate, q M stb/d	Rate, q MM stb/yr))	Cumulative N_p , M Stb
1996	0	-	-	-
1997	1	71.00	26,480.63	26,480.63
1998	2	55.56	20,293.29	46,773.92
1999	3	45.45	16,600.61	63,374.53
2000	4	38.46	14,047.52	77,422.05
2001	5	33.33	12,173.78	89,595.83
2002	6	29.41	10,742.00	100,337.83
2003	7	26.32	9,613.38	109,951.21
2004	8	23.81	8,696.60	118,647.81
2005	9	21.74	7,940.54	126588.35
2006	10	20.00	7,305.00	133,893.35
Total Oil Production in ten years is			138,795 M stb	

[Source: Generated using Table 3.8]

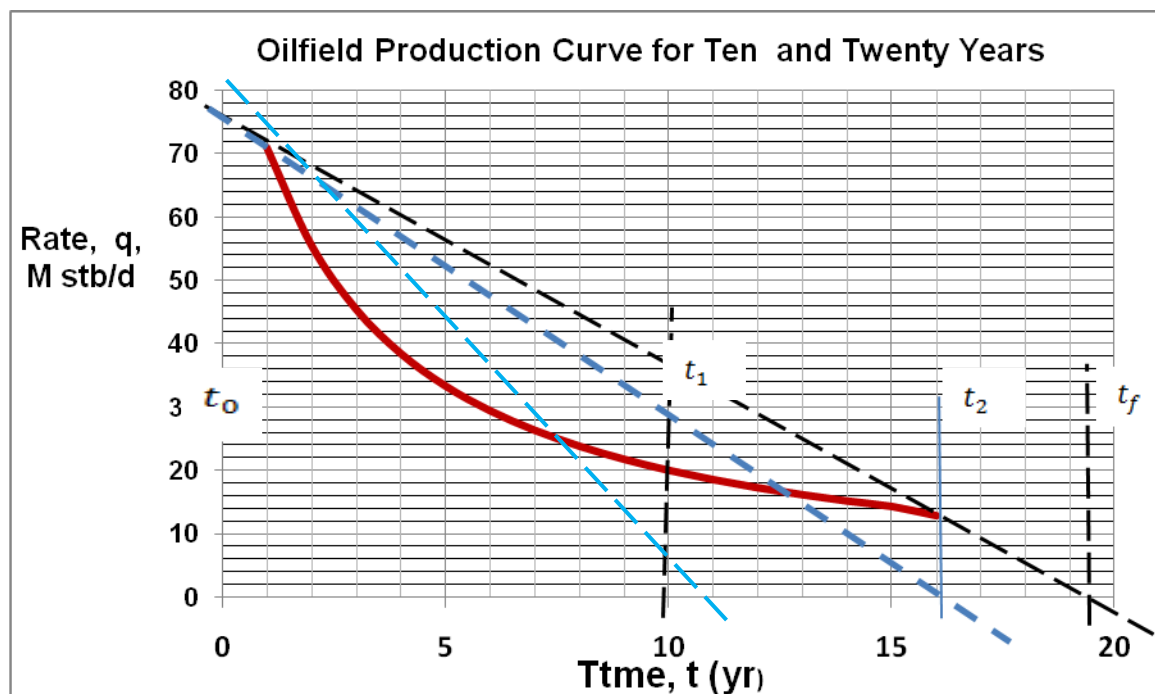


Fig 3.8: Curve Generated from the Projected Rate or Data on Table 3.8.

a. ESTIMATION OF THE HYDROCARBONS RECOVERY:

Solution from the curve showed that the rate, $q_1 = 73.5 \text{ stb/d}$, $t_0 = 0$, $t_1 = 10 \text{ yrs}$ and $t_f = 19.5 \text{ yrs}$. Putting these values in the model Eqn3.25, the cumulative oil production (N_p) was estimated and in Eqn3.26 oil initially in place was estimated as follows:

$$i. N_p = \frac{365.25 \cdot 73.5}{2} [10 - 0] = 134,229 \text{ M stb and } N_p = \frac{365.25 \cdot 20}{2} [19.5 - 10] = 34,699 \text{ M stb}$$

If the operator had maintained the reservoir pressure early enough, say from ten years up to 19.5 years the total oil recovery factor would have been improved from 38% to 73.5% as shown in solution-III, subsection - d below. $N_p = \frac{365.25 \cdot 73.5}{2} [19.5 - 0] = 261,747 \text{ M stb}$

b. ESTIMATION OF THE HYDROCARBONS INITIALLY IN PLACE:

The curve was extrapolated from time t_1 to time, t_f to estimate the hydrocarbons (Oil) initially in place using the model Eqn3.2.6. $N = \frac{365.25 \cdot 100}{2} [19.5 - 0] = 356,119 \text{ M stb}$

THE HYDROCARBONS RECOVERY FACTOR (E_R)

Recovery factor is the ratio of the cumulative hydrocarbon production to the hydrocarbon initially in place.

$$\text{Mathematically: } E_R = \frac{100N_p}{N} = \frac{100 \cdot 134,229}{356,119} = 38\% \quad E_R = \frac{100N_p}{N} = \frac{100 \cdot 261,747}{356,119} = 73.5\%$$

If the reservoir pressure were maintained the recovery factor would have been improved as shown below. Economic evaluation in this case would be the best method to enhance pressure maintenance consideration.

$$E_R = \frac{100N_p}{N} = \frac{100 \cdot 261,747}{356,119} = 73.5\%$$

RESULTS AND DISCUSSIONS:

RESULTS:

EVALUATION MODEL – 1: Figure 4.1 shows schematic of oil cumulative production and oil initially in place for projectile oil flow regime while Table 4.1 shows the confirmed projectile evaluation models.

Evaluation Model – 2: Figure 4.2 shows schematic of Oil-cumulative production and Oil initially in place for parabolic fluid flow regime, while Table 4.2 shows the confirmed parabolic fluid flow regime evaluation models oil flow.

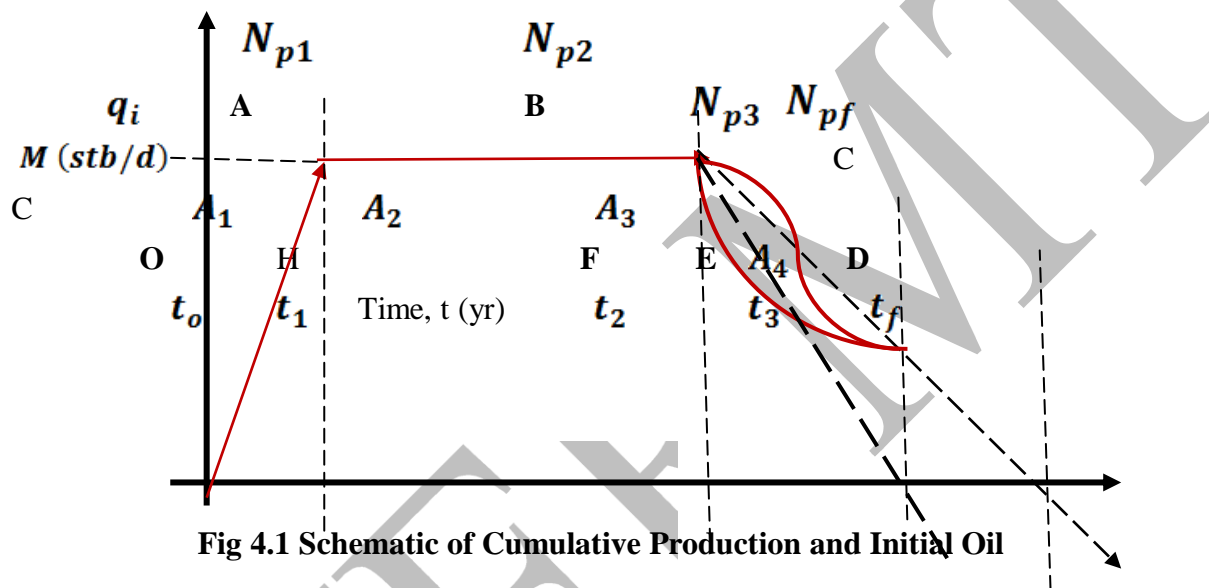


Fig 4.1 Schematic of Cumulative Production and Initial Oil

TABLE 4.1: CONFIRMED PROJECTILE EVALUATION MODELS

Type	Eqn	Flow Models	Remarks
Projectile Oil Flow Models	3.7	$b = \frac{\ln(q_i/q)}{t - t_i} \quad \text{For } n = 1$ $b = \frac{q_i - q}{q t} \quad \text{For } n = 2$	Rate Decline Constant
	3.20		$N_p = \frac{q_i}{2} [(t_2 - t_1) - (t_3 - t_0)]$
	3.11	$N = \frac{q_i}{2} [(t_2 - t_1) + (t_f - t_0)]$	Oil Initially in Place, M stb, Fig 4.1

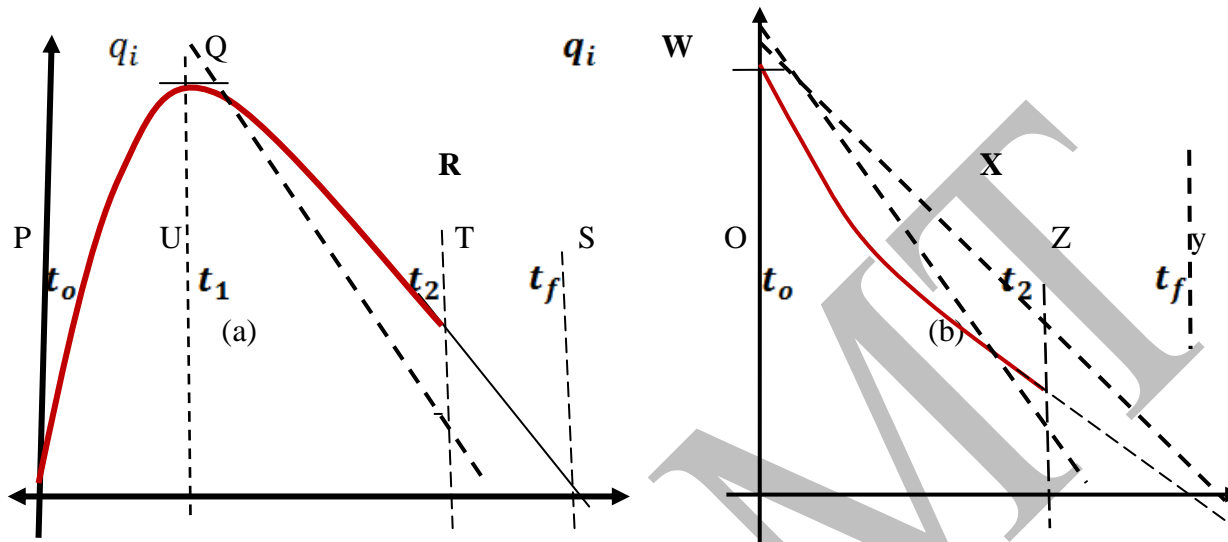


Fig 4.2 Cumulative Production and Initial Oil or Gas in Parabolic Flow Trend

Table 4.2: Confirmed Parabolic Evaluation Models – I

Type	Eqn	Model Equations	Remarks
Decline Rate for (n = 1)	3.15	$q = q_i e^{-bt}$ and $b = \frac{\ln(q_1/q_2)}{t_2 - t_1}$	First Order Oil flow Regime Models
	3.23	$N_p = \frac{q_i}{2} [(t_1 - t_0) + (t_2 - t_1)]$	
	3.24	$N = \frac{q_i}{2} [(t_1 - t_0) + (t_f - t_1)]$	
Decline Rate for (n = 2)	3.17	$q = \frac{q_i}{1 + bt}$ and $b = \frac{q_i - q}{q \Delta t} = \frac{q_1 - q_2}{q_2 t_2 - q_1 t_1}$	Second Order Oil flow Regime Models
	3.23	$N_p = \frac{q_i}{2} [(t_1 - t_0) + (t_2 - t_1)]$	
	3.24	$N = \frac{q_i}{2} [(t_1 - t_0) + (t_f - t_1)]$	
Decline Rate for fractions (n < 1) or (n < 2)	3.21	$q = \frac{q_i}{1 + bt}$ (i = 0, 1, 2, 3, ..., n)	Fractions flow Models
	3.22	$b \approx \sum_{i=1}^n \left[\frac{q_i - q_{i+1}}{q_{i+1} * t_{i+1}} \right]$ Meaning:	
	3.23	$b \approx \frac{1}{n} \left[\frac{q_i - q_1}{q_1 t_1} + \frac{q_i - q_2}{q_2 t_2} + \dots + \frac{q_i - q_n}{q_n t_n} \right]$	
	3.24	$N_p = \frac{q_i}{2} [(t_1 - t_0) + (t_2 - t_1)]$ $N = \frac{q_i}{2} [(t_1 - t_0) + (t_f - t_1)]$	
For Easy Unit Time Conversion		$e^{-bt} = (1 - b)^t$ $(1 - b/yr) = (1 - b/m)^{12} = (1 - b/d)^{365.25}$ $b/yr = 12 * b/m = 365.25b/d$	Ref: Taylor's Expansion

TABLE 4.3: CONFIRMED PARABOLIC EVALUATION MODELS – II

Type	Eqn	Model Equations	Remarks
Decline Rate for (n = 1)	3.7 3.25	$q = q_i e^{-bt}$ and $b = \frac{\ln(q_1/q_2)}{t_2 - t_1}$ $N_p = \frac{q_i}{2} [t_1 - t_0]$ and $N = \frac{q_i}{2} [t_f - t_0]$	First Order Oil flow Regime Models
Decline Rate for (n = 2)	3.21 3.25	$q = \frac{q_i}{1 + bt}$ and $b = \frac{q_i - q}{q \Delta t} = \frac{q_1 - q_2}{q_2 t_2 - q_1 t_1}$ $N_p = \frac{q_i}{2} [t_1 - t_0]$ and $N = \frac{q_i}{2} [t_f - t_0]$	Second Order Oil flow Regime Models
Decline Rate for fractions (n < 1) or (n < 2)	3.21 3.22 3.25	$q = \frac{q_i}{1 + bt}$ (i = 0, 1, 2, 3, ..., n) $b \approx \sum_i^n \left[\frac{q_i - q_{i+1}}{q_{i+1} * t_{i+1}} \right]$ Meaning: $b \approx \frac{1}{n} \left[\frac{q_i - q_1}{q_1 t_1} + \frac{q_i - q_2}{q_2 t_2} + \dots + \frac{q_i - q_n}{q_n t_n} \right]$ $N_p = \frac{q_i}{2} [t_1 - t_0]$ and $N = \frac{q_i}{2} [t_f - t_0]$	Oil Order of Flow Regime in Fractions Models

DISCUSSIONS:

PROJECTILE DOMINATED FLUIDS FLOW REGIME:

An oil reservoir production performance naturally results into a projectile flow trend when both the internal and external energies control the flow trend. This delays the boundary conditions since the reservoir principal mechanism at the early stage is an external energy drive system. That is possible, because the production rate increased in the initial stage from minimum to a peak value in a given time. On a peak value the rate was steady for another given time called (plateau/steady stage. After the peak value the transition or critical stage was observed due to boundary condition effects. The rate decline follows when the boundary effects seize and the internal energy takes over, from the peak value towards the economic flow rate value called an abandonment flow rate. The decline trend was classified into three main orders the first-order, second-order and fraction-order. Third order equations are mainly wave propagation and are very rare, so this research work does not cover the third order equations. In the third order equations, the wave tends to undergo simple harmonic motion (SHM) and most SHM tend to damped oscillation. The SHM is defined by the equation, $y = ax^n + bx + c$, with the solution as: $x = \frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. When the value of $b^2 - 4ac$ is negative, meaning that $b^2 < 4ac$ the flow equation $y = asinx + bcosx + c$ is perpetually observed, which is not common in the oil and/or gas fluid dynamics. Most projectile fluid dynamics or flow commonly tends to 1st order equation especially gas stream flow regimes, because the stability in the gas stream and 1st or 2nd order equation of fluid dynamics for oil, because of the instability in the flow and unsteady decline of the internal energy system of the reservoir. This is best explained when the external energy influence on the decline trend is negligible. In this case the transient flow period was long, but the steady state was longer. That was possible because the reservoir was saturated, with aquifer which could dominate initial state flow. It started by building up the internal energy for some time from time, t_0 to time, t_1 in fig 4.1. The steady state flow started from time, t_1 to time, t_2 , after that was the rate decline state, from time, t_2 to time, t_3

covering the cumulative oil recovery value (stb). Any un-recovery fluid from time, t_3 to time, t_f was the hydrocarbons supposed to be the residual oil saturation in that reservoir. The oil-reserves initially in place was estimated from time, t_o to time, t_f , using equation of the area of the flow trend. The value was confirmed using the field estimated value (Fig 4.1). Table 4.1 shows the evaluation models for projectile fluids dynamics.

PARABOLIC FLUID FLOW REGIME:

A hydrocarbon reservoir production performance naturally results into a parabolic dominated flow trend if both internal and external energies control the flow trend (Fig 4.2a), the transient stage is present, but plateau is absent. In Fig 4.2b, there is little or no external energy, so in a short period of production the boundary conditions effects influence the flow. The principal flow mechanism is an internal energy drive system only, so decline sharply after a short time of production..The transition stage would be sharp, very short or not noticeable in some plots. The production rate decline trend in parabolic dominated flow trend is classified into three main orders the first-order where the decline exponent is unity ($n = 1$) with fairly steady decline constant ‘b’,, second-order where the decline constant ‘b’ is fairly steady as well, but decline exponent is two ($n = 2$) and fraction orders ($n < 1$ or $1 < n < 2$), where the production decline rate value ‘b’ is not constant. When the order is either less than one or less than two ($1 < n < 2$), the production rate decline tended to increase from the initial stage (peak value) towards the minimum value of the reserves or sharply changed to a decline production rate in a short time. The plateau or transient stage would seem to be absent and the transition stage would not be noticeable. The reservoir flow declines from the peak value towards the economic flow rate value called an abandonment flow rate. If $b_1 = b_2 = b_3 = \dots = b_n$ and $n = 1$, it implies uniform decline. If $b_1 = b_2 = b_3 = \dots = b_n$ and $n = 2$, it indicates uniform decline. When $n < 1$ or $1 < n < 2$, the value of, $b_1 \neq b_2 \neq b_3 \neq \dots \neq b_n$, it indicates non-uniform decline rate. In this case an average decline would be used.

APPLICATION OF THE EVALUATION MODELS USING GENERIC DATA

The advantage in using generic data is mainly to enhance hydrocarbons production projected values. This makes it easy to predict future hydrocarbons production performances and take decision on the reservoir pressure management. The results showed high accuracy on the forecast. The percentage accuracy for oil ranged from 98.64% to 99.98%. Table 4.4 shows the comparison of the model results with the tanks, tabulated and Craze - Buckley MBE estimated values, while and Table 4.5 shows a comparison of the models and tabulated values.

Table 4.4: The Model Results for Oil Compared with the Tank and MBE Values

S/No	Values Used	$N_p, MMstb$	Accuracy	$N, MMstb$	Accuracy
1.	Models	16.88	98.64%	20.27	99.30%
2.	Tanks	16.70			
3.	Tabulated Tables	16.65			
4.	Craze and Buckley MBE	16.65		20.12	

Table 4.5: Model Results Using Generic Data Compared with Tabulated Oil Values

S/No	Values Used	$N_p, Mstb$	% Accuracy	N, Mstb	% Accuracy
1	Models	68.50	99.77%	105.92	-
	Tabulated Values	68.34	-	-	-
2	Models	136.96	99.97%	438.30	-
	Tabulated Values	137.00	-	-	-

3	Models Tabulated Values	134.23 133.90	99.75%	356.12 -	-
---	----------------------------	------------------	--------	-------------	---

Source [Model and Tabulated Results]

CONCLUSION AND RECOMMENDATIONS:**CONCLUSION:**

Mathematical models equations were successfully derived for studying reservoirs fluids depletion from the peak value at decline stage to an economic value called abandonment. Decline rate trends analysis showed two types of flow projectile dominated flow regimes attributed to external, internal energies and boundary conditions effects and Parabolic flow regimes whose principal mechanism is due to internal and boundary conditions effects. The projectile dominated flow models were mainly used to generate curves for predicting hydrocarbons production performances. When the reservoir pressure is above its bubble-point pressure, projectile dominated flow is possible and evaluation models-I should be used, but when the reservoir pressure is closed or at bubble-point pressure, the parabolic dominated flow is possible in that well and evaluation models-II should be used. This is because the bubble point pressure is the critical point for critical rate. Highly above the bubble point the dominated fluids flow is the projectile type, while slightly above the

bubble point pressure down to the abandonment point parabolic dominated fluid flow regime is expected. The parabolic dominated fluid flow models were used to predict future recovery from the decline stage to an economic rate (abandonment). The extrapolation of the curve from the decline point to the economic rate point on the t-axis at t_f gave the total reserves in place.

OBSERVATIONS:

When yearly rates projected to abandonment or close to it are used to generate curves, the models give high accuracy estimated reserves (N_p and N). When first and last points of the decline stages are extrapolated for actual flow rate (q) and time (t), as the models input data, they give high accuracy estimated reserves (N_p and N). **Yearly** rates and pressure depletion trend synergy was necessary to predict transient and steady states periods, but was not used here.. Projected production performances of reserves and estimation of the reserves initially in place percentage accuracy for gas fields ranged from 99.86% and above, while the percentage accuracy for oil ranged from 98.64% to 99.98%.. This enhances proper reservoir pressure planning and management for high oil recovery. The result of this research simplifies complex simulation methods, improves DFCA accuracy and makes it easy to identify dominated flow and rates decline trends.

RECOMMENDATIONS:

- First and last points of the decline stage must be extrapolated to the axes in order to obtain actual flow rate, q and time, t as the model input. This gives high estimation accuracy.
- Only projected rates to abandonment stage close to it should be used to estimate reserves. It improves reserves estimation accuracy.
- Yearly rates and pressure decline synergy is not used and production depends on pressure sustainability. Hence it is recommended that pressure maintenance should be used (if required) to manage the reservoir pressure for economic recovery.

NOMENCLATURE:

A: Area of the reservoir, *acres or ft²*

a : Actual decline fraction of production rate

α_1 : = Initial oil or gas production decline

AGBADA FORMATION:

Geological formation which consists mainly of sandstones, shale alternation with the sandstones predomination

Akata Formation: This is a Marine pro-Delta mainly shale-stones and siltstones, which crop out in sub-sea outer Delta.

AGA: American Gas Association, generally acceptable standard units

°API: Oil or Gas Gravity, API (American Petroleum Institute)

b: Rate Decline Constant, *Mbbl/yr or Mbbl/d*

Bbl: Barrel (Unit of oil or liquid measurement)

Benin formations: This is mainly sand and sandstones, coarse to fine, granular in texture and partly unconsolidated formation.

B_{of}: Actual oil formation volume factor,

B_{oi}: Initial Oil formation volume factor, *rb/stb*

Bubble Point Pressure: Critical pressure condition for rate decline initiation

CAPEX: Capital Expenses (Development Bills)

D: Depth of the reservoir, *ft*

DCA (Decline Curve Analysis): Mathematical equations, tabulated tables or graphical procedures for studying the oil and/or gas production rate, prediction of cumulative oil or projected oil production

Decline Curve (Tend): Graphical representation of oil or gas production rate

Decline Rate: Reduction of a production volume per unit time, *Mbbl/yr*

DPR: Department of Petroleum Resources, NNPC subsidiary

DFCA: Dynamic Fluids Computational Analysis

E_i: Gas expansion factor, %

G: Gas initially in place (GIIP), MMscf

G_p: Cumulative gas recovery in a reservoir, MMscf

GOR: Gas - Oil Ratio, *scf/bbl*

MBE: Materials Balance Equation (quick volume changes estimation)

n: Rate Decline exponential or production decline rate power constants

N: Oil initially in place (OIIP), *stb*

NNPC: Nigerian National Petroleum Cooperation (Oil / Gas operation Age)

N_p: Cumulative oil production in a reservoir, *stb*

OPEX: Operations Expenses (Daily operation costs or bills)

q: Actual hydrocarbons flow rate, *bbl/d or bbl/yr*

q_i : Initial oil or gas production flow rate, *bbl/d or bbl/yr*

STB or stb: stalk tank barrel

SCF or scf: Standard cubic feet, *ft³*

t : Time unit (s, hr or yr)

Transient Part or Stage: Unsteady rate in the initial stage of production

Transition Stage: A critical stage which could result into a decline stage

t_o, t₁, t_f: Unit time, *ft*

γ_g: Gas specific gravity, dimensionless

Z: Gas deviation or compressibility factor, %

$\frac{1}{n} = h$ = hyperbolic decline constant

REFERENCES:

1. **Arps, J. J. (1945)** "Analysis of Decline Curve" Trans, AIME, Vol. 160 PP228-247
2. **Craft, B. C. and M. F. Hawkins (1959)**, "Applied Petroleum Reservoir Engineering" Englewood Cliffs, NJ, Chapter- 9 and 10: PP203 - 265.
3. **Katz, D. L., (1959)** "Handbook of Natural Gas Engineering" McGraw-Hill, Inc., New York.
4. **Arps, J.J. and T.C. Frick (1962)**, "Petroleum Production Handbook Volume-XI", Chapter-7 (Frick T, C, Ed Richardson, TX, SPE AIME, 1962) P37
5. **Edwardson, E. R., (1962)** "Mathematical Equations for Cumulative Production in a Decline Rate Analysis" J. Pet. Tech. 1962
6. **Bruns, J. R., Fetkovitch, M. J. and V. C. Meitzen (1965)** "The Effects of Water Influx on P/Z Cumulative Gas Production Curves" J-Pet Tech March, 1965. PP287-291
7. **Bailey, D. L., (1982)** "Decline Rate in Fractured Gas Wells" OGJ, 15th Feb., 1982. PP117 -118
8. **Fetkovitch, M. J., (1984)** "Advanced Decline Curve Analysis Approach", J. Pet. Tech.
9. **Blasigame, R., McClay and Palacio, J., (1989)** "Rate Integral Function and Rate integral Derivatives Function types Curve Analysis" J. Pet Tech.
10. **Economides, M. J., Hills, A. D. and Ehilg-Economides, C., (1994)** "Petroleum Production System" Upper Saddle River, NJ; Prentice Hull PTR, 1994. PP500-520
11. **Ramsey, H. J. (Jr.) and Guerror, E. T. (Jr.), (2002)** "Relative Oil Decline Rate Analysis" Upper Saddle River, NJ; Prentice Hull PTR, 2002. PP150-158
12. **King Hubbert, I. P and Robertson, J W. (2004)** "Modified Hyperbolic Decline" J. Pet Tech, Nov., 2004. PP21-29
13. **Amini, R. T., Mc-Cray and Palacio, D. C. (2007)** "Governing Flow Regime for Low Permeability Reservoirs" Upper Saddle River, NJ; Prentice Hull PTR, 2007. PP247-258
14. **Agarwal, G. E and Gardner, T. A. (2008)** "Oil Production Decline Types Curves for Analyzing Production Data" Upper Saddle River, NJ; Prentice Hull PTR, 1994. PP'250-273
15. **Ilk, E. T. (2008)**, "Power Law Decline Method" OGJ, 2008.
16. **Obah, B., Aniefiok, L. and Ezugwu, C. M., (2012)** "Simplified Models for Forecasting Oil Production: Niger Delta Oil Rim Reservoirs" Petroleum Technology Development Journal (ISSN1595-9104): An International Journal; July, 2012 – Vol-2; No. 2